

一种新型 Hopfield 神经网络平衡解的全局指数稳定性

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摘 要: 结合生物学最近的研究成果, 提出一种具有动态突触和不同时间尺度的新型 Hopfield 神经网络模型. 运用不动点定理、不等式技巧及 Lyapunov 泛函等方法, 讨论了该人工神经网络在具有常时滞和变时滞两种情况下, 其平衡解的存在性唯一性和全局指数稳定性.

关键词: 动态突触; 时间尺度; 神经网络; 指数稳定

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Globally Exponential Stability of Equilibrium for New Hopfield Neural Networks

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Abstract: A new Hopfield neural networks with dynamical synapses and multiple time scales is proposed in this paper, based on some recent progresses in biological science. Fixed point theorem, inequality technique and Lyapunov functional methods have been used in this paper to study properties such as the global exponential stability, existence and uniqueness of the equilibrium solution for both constant and variable time delays.

Key words: dynamic synapses; time scales; neural networks; exponential stability

1 引言

生物学方面取得的成果对神经网络的发展有着深远的影响. 近几年, 神经生物学方面有两个进展值得关注: 一是神经元之间的突触是动态变化的, 从而神经网络的连接权也应该是动态变化的^[1-4]; 二是人类的记忆方式有长期记忆 (LTM) 和短期记忆 (STM) 之分, 不同的记忆方式其内在的生物学机制也不同, 体现在数学上就是其动力学方程不一样^[5,6]; 因此在设计神经网络时应区别对待. Hopfield 神经网络是一类重要的神经网络, 受到广大学者的关注^[9-12]. 物理学家 Hopfield 提出这个模型并用电子电路对生物神经网络进行简单的模拟, 并把外部刺激 (电流) 设为常数; 后来的文献多把 I_i 当作干扰项而处理掉. 事实上, 根据文献^[5]的研究结果, 外部刺激是 LTM 产生的根源, 是整个神经网络不可或缺的一部分; 并且考虑了一个神经元同一时刻有多个外部刺激的现象.

引入文献^[2]中神经元突触变化的动力学模型以及文献^[5]中关于外部刺激的动力学模型; 另外, 从生物学的观点出发, 突触资源的变化还要受到神经元本身状态的影响; 此外, 时滞因素的影响也是不可忽略的. 考虑如下 Hopfield 神经网络

$$\text{STM: } \dot{x}_i(t) = -b_i x_i(t) + \sum_{j=1}^n w_{ij} f_j(t) f_j(x_j(t - \tau_j)) + B_i s_i(t - \tau_{i+2n})$$

$$\text{LTM: } \dot{s}_i(t) = -s_i(t) + f_i(x_i(t - \tau_i)) \quad (1)$$

$$\dot{r}_i(t) = -c_i r_i(t) + u_i(1 - r_i(t)) g_i(x_i(t - \tau_i))$$

其中, $b_i > 0$, $c_i > 0$, $x_i(t)$ 为第 i 个神经元的状态变量, $r_j(t)$ 为第 j 个神经元的可用突触资源, w_{ij} 为静态连接权 (常数), $f_j(x_j(t))$ 是第 j 个神经元的传递函数, $g_j(x_j(t))$ 是第 j 个神经元对 $r_j(t)$ 的效率影响函数, $u_i > 0$ 是突触资源的效率常数, $s_i(t)$ 是经过技术处理后的外部刺激变量, $B_i > 0$ 是其相应的作用常数.

2 定义及引理

为讨论方便, 设系统(1)的初始函数为 $x_i(t) = \varphi_i(t)$, $r_i(t) = \varphi_{i+n}(t)$, $s_i(t) = \varphi_{i+2n}(t)$, $t \in [-\tau, 0]$, $i = 1, 2, \dots, n$, 其中 $\tau = \max_i \tau_i$, 并设 $\|\varphi\| = \max_{i, t \in [-\tau, 0]} (|\varphi_i(t)|, |\varphi_{i+n}(t)|, |\varphi_{i+2n}(t)|)$.

定义 系统(1)的平衡解 $(\bar{x}_1, \dots, \bar{x}_n, \bar{r}_1, \dots, \bar{r}_n, \bar{s}_1, \dots, \bar{s}_n)$ 称为全局指数稳定的, 若对 $\forall \delta > 0$ 存在一列正常数 $\rho_i(\delta)$, $\rho_{i+n}(\delta)$, $\rho_{i+2n}(\delta)$ 及 $\eta > 0$, 使得系统(1)的任意解当 $\|\varphi\| \leq \delta$ 时满足: $|x_i(t) - \bar{x}_i| \leq \rho_i(\delta) e^{-\eta t}$, $|r_i(t) - \bar{r}_i| \leq \rho_{i+n}(\delta) e^{-\eta t}$, $|s_i(t) - \bar{s}_i| \leq \rho_{i+2n}(\delta) e^{-\eta t}$, $t \geq 0, i = 1, 2, \dots, n$.

引理 1 若系统(1)满足下列条件

$$(T) \quad \forall a, b \in R, |g_i(a)| \leq L_i, \\ |f_i(a)| \leq M_i, |f_i(a) - f_i(b)| \leq \alpha_i |a - b|, \\ |g_i(a) - g_i(b)| \leq \beta_i |a - b|, c_i > u_i L_i.$$

则系统(1)的解 $(x(t), s(t), r(t)) (t \geq 0)$ 最终有界, 其中 $\tau = \max_i \tau_i$.

证明 系统(1)的第二个方程变形为

$$\dot{r}_i(t) = -[c_i + u_i g_i(x_i(t - \tau_i))] r_i(t) + u_i g_i(x_i(t - \tau_i)), \\ i = 1, 2, \dots, n$$

由条件(T)知, $[c_i + u_i g_i(x_i(t - \tau_i))] \geq [c_i - u_i L_i] > 0$, 且有 $-[c_i + u_i g_i(x_i(t - \tau_i))] r_i(t) - u_i L_i \leq -[c_i + u_i g_i(x_i(t - \tau_i))] r_i(t) + u_i L_i$, 当 $t > 0$ 时, 由比较原理可证明

$$r_i(t) \leq \varphi_i(0) \exp \left[\int_0^t -[c_i + u_i g_i(x_i(s - \tau_i))] ds \right] \\ + \int_0^t u_i L_i \exp \left[- \int_0^s [c_i + u_i g_i(x_i(y - \tau_i))] dy \right] ds \\ \leq |\varphi_i(0)| \exp \left[\int_0^t -[c_i - u_i L_i] ds \right] \\ + \int_0^t u_i L_i \left[\exp \left[- \int_s^t [c_i - u_i L_i] dy \right] \right] ds \\ = |\varphi_i(0)| \exp(-[c_i - u_i L_i] t) \\ + \frac{u_i L_i}{c_i - u_i L_i} [1 - \exp(-(c_i - u_i L_i) t)]$$

从而 $\lim_{t \rightarrow +\infty} r_i(t) \leq \frac{u_i L_i}{c_i - u_i L_i} \triangleq R_i$; 类似, $\lim_{t \rightarrow +\infty} r_i(t) \geq -\frac{u_i L_i}{c_i - u_i L_i}$; 从而 $r_i(t) (t \geq 0)$ 最终有界, 结合 $|f_i(a)| \leq M_i$, 易证 $x_i(t), s_i(t) (t \geq 0, i = 1, 2, \dots, n)$ 也最终有界.

3 主要结论

定理 1 若系统(1)满足条件(T), 则系统(1)存在平衡点.

证明 只证下列方程组有解即可.

$$\begin{cases} -b_i x_i + \sum_{j=1}^n w_{ij} f_j(x_j) + B_i s_i = 0 \\ -c_i x_i + u_i(1 - r_i) g_i(x_i) = 0 \end{cases} \\ \Rightarrow x_i = b_i^{-1} \left[\sum_{j=1}^n w_{ij} \frac{u_j g_j(x_j)}{c_j + u_j g_j(x_j)} f_j(x_j) + B_i f_i(x_i) \right] \\ -s_i + f_i(x_i) = 0 \\ \text{令 } F_i(x) = b_i^{-1} \left[\sum_{j=1}^n w_{ij} \frac{u_j g_j(x_j)}{c_j + u_j g_j(x_j)} f_j(x_j) + B_i f_i(x_i) \right], \\ x = (x_1, \dots, x_n), i = 1, 2, \dots, n$$

则

$$|F_i(x)| \leq b_i^{-1} \left[\sum_{j=1}^n \left| w_{ij} \frac{u_j g_j(x_j)}{c_j + u_j g_j(x_j)} f_j(x_j) \right| + B_i |f_i(x_i)| \right] \\ \leq b_i^{-1} \left[\sum_{j=1}^n |w_{ij}| \frac{u_j L_j}{c_j - u_j L_j} M_j + B_i M_i \right]$$

令 $M = \max_i b_i^{-1} \left[\sum_{j=1}^n |w_{ij}| \frac{u_j L_j}{c_j - u_j L_j} M_j + B_i M_i \right]$, 则 $A = \{x \in R^n | |x_i| \leq M\}$ 是有界凸闭集, 显然 F 是连续的, 故由 Brouwer 不动点定理^[13], $F(x)$ 在 A 上至少有一个不动点; 不妨设一平衡点为 $(\bar{x}_1, \dots, \bar{x}_n)$, 令 $\bar{s}_i = f_i(\bar{x}_i)$, $\bar{r}_i = \frac{u_i g_i(\bar{x}_i)}{c_i + u_i g_i(\bar{x}_i)}$, $i = 1, 2, \dots, n$, 显然, $(\bar{x}_1, \dots, \bar{x}_n, \bar{r}_1, \dots, \bar{r}_n, \bar{s}_1, \dots, \bar{s}_n)$ 是系统(1)的一个平衡点; 证毕

设 $(x_1(t), \dots, x_n(t), r_1(t), \dots, r_n(t), s_1(t), \dots, s_n(t))$ 是系统(2)的任一解, 令

$$y_i(t) = x_i(t) - \bar{x}_i, y_{i+n}(t) = r_i(t) - \bar{r}_i, \\ y_{i+2n}(t) = s_i(t) - \bar{s}_i, h_i(y_i(t)) = f_i(x_i(t)) - f_i(\bar{x}_i), \\ h_{i+n}(y_{i+n}(t)) = g_i(x_i(t)) - g_i(\bar{x}_i), i = 1, 2, \dots, n$$

考虑如下系统

$$\begin{aligned} \dot{y}_i(t) &= -b_i y_i(t) + \sum_{j=1}^n w_{ij} [r_j(t) (h_j(y_j(t - \tau_j)) \\ &\quad + f_j(\bar{x}_j) y_{j+n}(t))] + B_i y_{i+2n}(t - \tau_{i+2n}) \\ \dot{y}_{i+n}(t) &= -c_i y_{i+n}(t) - u_i g_i(\bar{x}_i) y_{i+n}(t) \\ &\quad u_i [1 - r_i(t)] h_{i+n}(y_{i+n}(t - \tau_i)) \\ \dot{y}_{i+2n}(t) &= -y_{i+2n}(t) + h_i(y_i(t - \tau_i)) \end{aligned} \quad (2)$$

显然, 系统(1)的平衡解的全局吸引性等价于(2)的零解全局吸引性. 设

$$C_1 = \text{diag}(b_1, b_2, \dots, b_n), \\ C_2 = \text{diag}(c_1 - u_1 L_1, c_2 - u_2 L_2, \dots, c_n - u_n L_n), \\ C_3 = E, \\ D_0 = \begin{bmatrix} |w_{ij}| M_j \end{bmatrix}_{n \times n}, \\ D_1 = \begin{bmatrix} |w_{ij}| R_j \alpha_j \end{bmatrix}_{n \times n}, \\ D_2 = \text{diag}(u_1(1 + R_1) \beta_1, u_2(1 + R_2) \beta_2, \dots, u_n(1 + R_n) \beta_n), \\ D_3 = \text{diag}(\alpha_1, \alpha_2, \dots, \alpha_n), \\ D_4 = \text{diag}(B_1, B_2, \dots, B_n).$$

假设

$$C = \begin{pmatrix} C_{ij} \end{pmatrix}_{3n \times 3n} = - \begin{pmatrix} C_1 & & \\ & C_2 & \\ & & C_3 \end{pmatrix},$$

$$D = \begin{pmatrix} D_{ij} \end{pmatrix}_{3n \times 3n} = \begin{pmatrix} D_1 & 0 & D_4 \\ D_2 & 0 & 0 \\ D_3 & 0 & 0 \end{pmatrix},$$

$$\hat{D} = \begin{pmatrix} \hat{D}_{ij} \end{pmatrix}_{3n \times 3n} = \begin{pmatrix} 0 & D_0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

定理 2 若系统(1) 满足(T), 且 $-(C+D+\hat{D})$ 是 M-矩阵, 则系统(1) 的平衡解唯一且全局指数稳定.

证明 由引理 1 知, 对任意的初始条件, 存在 $T > 0$, 当 $t \geq T$ 时, $r_i \leq R_i$, 本文以后讨论系统(1) 的解时如不特殊说明均指 $r \geq T$ 的情况; 由系统(2) 的第一个方程可得

$$D^+ |y_i(t)| \leq -b_i |y_i(t)|$$

$$+ \left| \sum_{j=1}^n w_{ij} [r_j(t)(h_j(y_j(t-\tau_j)) + f_j(\bar{x}_j))y_{j+n}(t)] \right|$$

$$+ |B y_{i+2n}(t-\tau_{i+2n})|$$

$$\leq -b_i |y_i(t)|$$

$$+ \sum_{j=1}^n |w_{ij}| [R_j \alpha_j |y_j(t-\tau_j)| + M_j |y_{j+n}(t)|]$$

$$+ B_i |y_{i+2n}(t-\tau_{i+2n})|$$

同理

$$D^+ |y_{i+n}(t)| \leq -[c_i - u_i L_i] |y_{i+n}(t)|$$

$$+ u_i [1 + R_i] \beta_i |y_i(t-\tau_i)|$$

$$D^+ |y_{i+2n}(t)| \leq -|y_{i+2n}(t)| + \alpha_i |y_i(t-\tau_i)|$$

综合可得

$$D^+ |y_i(t)| \leq C_{ii} |y_i(t)| + \sum_{j=1}^{3n} D_{ij} |y_j(t-\tau_j)|$$

$$+ \sum_{j=1}^{3n} \hat{D}_{ij} |y_j(t)|, \quad i = 1, 2, \dots, 3n, t \geq T \quad (3)$$

因 $-(C+D+\hat{D})$ 是 M-矩阵, 由 M-矩阵的定义, 存在一列正数 $k_1, k_2, \dots, k_{3n}$, 使得 $k_i C_{ii} + \sum_{j=1}^{3n} k_j (D_{ji} + \hat{D}_{ji}) < 0$, 故

$$\text{存在 } \eta > 0, \text{ 使得 } k_i (C_{ii} + \eta) + \sum_{j=1}^{3n} k_j (D_{ji} e^{\eta \tau_j} + \hat{D}_{ji}) < 0, \quad i =$$

$$1, 2, \dots, 3n, \text{ 设 } z_i(t) = |y_i(t)| e^{\eta t}, \text{ 则}$$

$$D^+ z_i(t) \leq \eta e^{\eta t} |y_i(t)| + e^{\eta t} C_{ii} |y_i(t)|$$

$$+ e^{\eta t} \sum_{j=1}^{3n} (D_{ij} |y_j(t-\tau_j)| + \hat{D}_{ij} |y_j(t)|)$$

$$= (\eta + C_{ii}) z_i(t) + \sum_{j=1}^{3n} [D_{ij} e^{\eta \tau_j} z_j(t-\tau_j)$$

$$+ \hat{D}_{ij} z_j(t)]$$

若令 $V(t) = \sum_{i=1}^{3n} k_i \left[z_i(t) + \sum_{j=1}^{3n} \int_{t-\tau_j}^t D_{ij} e^{\eta \tau_j} z_j(s) ds \right]$, 则由

式(3) 可得

$$D^+ V(t)$$

$$\leq \sum_{i=1}^{3n} k_i \left[(C_{ii} + \eta) z_i(t) + \sum_{j=1}^{3n} (D_{ij} e^{\eta \tau_j} z_j(t) + \hat{D}_{ij} z_j(t)) \right]$$

$$= \sum_{i=1}^{3n} k_i (C_{ii} + \eta) z_i(t) + \sum_{j=1}^{3n} k_j (D_{ji} e^{\eta \tau_j} + \hat{D}_{ji}) z_j(t)$$

$$= \sum_{i=1}^{3n} k_i (C_{ii} + \eta) + \sum_{j=1}^{3n} k_j (D_{ji} e^{\eta \tau_j} + \hat{D}_{ji}) z_i(t)$$

$$\leq m \sum_{i=1}^{3n} z_i(t) \quad (4)$$

其中 $m = \max_i \sum_{j=1}^{3n} [k_i (C_{ii} + \eta) + \sum_{j=1}^{3n} k_j (D_{ji} e^{\eta \tau_j} + \hat{D}_{ji})] < 0$, 对

式(4) 从 T 到 t 积分可得, $V(t) - \int_T^t m \sum_{i=1}^{3n} z_i(s) ds \leq V(T)$

$\Rightarrow 0 \leq - \int_T^t m \sum_{i=1}^{3n} z_i(s) ds < \infty$. 注意到 $m < 0, z_i(t) \geq 0$, 故 $z_i(t)$ 在 $[T, +\infty)$ 上有界, 若令 $z_i(t) \leq \bar{\rho}_i$, 则 $|y_i(t)| \leq \bar{\rho}_i e^{-\eta t}, t \geq T, i = 1, 2, \dots, 3n$, 而在闭区间 $[0, T]$ 上 $|y_i(t)|$ 有界, 不妨设 $|y_i(t)| \leq K_i (\|\varphi\|)$, 令 $\bar{\rho} = \max_i (K_i (\|\varphi\|) e^{\eta T}, \bar{\rho}_i)$, 则在 $[0, +\infty)$ 上有 $|y_i(t)| \leq \bar{\rho}_i e^{-\eta t}$, 故平衡解的全局指数稳定性得证. 当 $t \rightarrow +\infty$ 时, $|y_i(t)| \rightarrow 0$, 平衡解唯一性可得. 证毕.

考虑更一般的情形, 假设系统(1) 中的时滞是变时滞, 即考虑如下系统

$$\text{STM: } \dot{x}_i(t) = -b_i x_i(t) + \sum_{j=1}^n w_{ij} r_j(t) f_j(x_j(t-\tau_j(t))) + B_i s_i(t-\tau_{i+2n}(t))$$

$$\text{LTM: } \dot{s}_i(t) = -s_i(t) + f_i(x_i(t-\tau_i(t))) \quad (5)$$

$$\dot{r}_i(t) = -c_i r_i(t) + u_i(1-r_i(t)) g_i(x_i(t-\tau_i(t)))$$

显然下列系统零解的指数稳定性等价于式(5) 的平衡解的指数稳定性

$$\dot{y}_i(t) = -b_i y_i(t) + \sum_{j=1}^n w_{ij} [r_j(t) h_j(y_j(t-\tau_j(t)))$$

$$+ f_j(\bar{x}_j) y_{j+n}(t) + B_i y_{i+2n}(t-\tau_{i+2n}(t))]$$

$$\dot{y}_{i+n}(t) = -c_i y_{i+n}(t) - u_i g_i(\bar{x}_i) y_{i+n}(t)$$

$$+ u_i [1 - r_i(t)] h_{i+n}(y_i(t-\tau_i(t)))$$

$$\dot{y}_{i+2n}(t) = -y_{i+2n}(t) + h_i(y_i(t-\tau_i(t))) \quad (6)$$

定理 3 如果系统(6) 满足条件(T) 且 $-(C+D+\hat{D})$ 是 M-矩阵, $0 \leq \tau_i(t) \leq \tau_i, \tau_i = \max_i \tau_i, \tau_i'(t) \leq 0, i = 1, 2, \dots, 3n$, 则系统(6) 的平衡解是唯一的且全局指数稳定.

证明 与定理 2 的证明类似, 可以得到如下结论

$$D^+ |y_i(t)| \leq C_{ii} |y_i(t)| + \sum_{j=1}^{3n} D_{ij} |y_j(t-\tau_j(t))|$$

$$+ \sum_{j=1}^{3n} \hat{D}_{ij} |y_j(t)|, \quad i = 1, 2, \dots, 3n, t \geq T$$

同样, 令 $z_i(t) = |y_i(t)| e^{\eta t}, z_i(t-\tau_i(t)) = |y_i(t-\tau_i(t))| e^{\eta(t-\tau_i(t))}$

$\tau_i(t)) | e^{\eta(t-\tau_i(t))} \geq |y_i(t-\tau_i(t))| e^{\eta(t-\tau_i(t))}$, 则

$$\begin{aligned} D^+ z_i(t) &\leq \eta e^{\eta t} |y_i(t)| + C_{ii} |y_i(t)| e^{\eta t} \\ &\quad + e^{\eta t} \sum_{j=1}^{3n} (D_{ij} |y_j(t-\tau_j(t))| + \hat{D}_{ij} |y_j(t)|) \\ &\leq (\eta + C_{ii}) z_i(t) + \sum_{j=1}^{3n} D_{ij} e^{\eta \tau_j(t)} z_j(t-\tau_j(t)) \\ &\quad + \sum_{j=1}^{3n} \hat{D}_{ij} z_j(t) \\ &\leq (\eta + C_{ii}) z_i(t) + \sum_{j=1}^{3n} D_{ij} e^{\eta \tau_j(t)} z_j(t-\tau_j(t)) \\ &\quad + \sum_{j=1}^{3n} \hat{D}_{ij} z_j(t) \end{aligned}$$

令 $V(t) = \sum_{i=1}^{3n} k_i [z_i(t) + \sum_{j=1}^{3n} \int_{t-\tau_j(t)}^t D_{ij} e^{\eta \tau_j(s)} z_j(s) ds]$, 注意到 $z_j \geq 0$ 和 $\tau_j(t) \leq 0$, 则有

$$\begin{aligned} D^+ \left[\sum_{j=1}^{3n} \int_{t-\tau_j(t)}^t D_{ij} e^{\eta \tau_j(s)} z_j(s) ds \right] \\ = \sum_{j=1}^{3n} [D_{ij} e^{\eta \tau_j(t)} z_j(t) - (1 - \tau_j'(t)) D_{ij} e^{\eta \tau_j(t-\tau_j(t))} z_j(t-\tau_j(t))] \\ \leq \sum_{j=1}^{3n} [D_{ij} e^{\eta \tau_j(t)} z_j(t) - D_{ij} e^{\eta \tau_j(t-\tau_j(t))} z_j(t-\tau_j(t))] \end{aligned}$$

$$\begin{aligned} D^+ V(t) \\ \leq \sum_{i=1}^{3n} k_i \left[(\eta + C_{ii}) z_i(t) + \sum_{j=1}^{3n} (D_{ij} e^{\eta \tau_j(t)} z_j(t) + \hat{D}_{ij} z_j(t)) \right] \end{aligned}$$

此时可证定理结论成立; 其过程与定理 2 类似, 略去. 证毕.

4 结束语

从生物学的角度出发, 讨论具有不同时间尺度以及动态突触的神经网络更有实际意义, 这方面的工作已受到广泛关注. 针对常时滞和变时滞的情况, 本文研究了一类具有上述特点的 Hopfield 神经网络平衡解的存在性及全局指数稳定性, 并得到了较易判别的代数方法, 便于实际工作者构造需要的人工神经网络.

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